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TERRAIN AND SEA SURFACE TRUTH: PROFILE DISTRIBUTIONS

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FOR THE COMMANDER

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The surface truth of particular interest in this paper is the surface profile distributions of height, slope, and base random variables averaged over the same or similar ensemble of paths above a terrain or sea surface from which coherent and incoherent scatter measurements are averaged. One theoretical scattering model, in which the surface height is assumed to be exponentially distributed, was recently shown in ESD-TR-81-147 (AD A106133) to be superior to other height distributions and to be at least as good as the Longley-Rice semi-empirical model in fitting three sets of measure- (OVER)

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ment data for microwave and acoustical coherent forward scatter from smooth and moderately rough surfaces. The question arises as to how well the assumed exponential distribution agrees with the actual distribution of surface heights.

This paper reviews the empirical data of surface profile distributions reported in the literature and particularly those associated with the above three sets of scattering data. Surface [illegible] distribution data was found to be sparce and inconclusive. A surface profile data base and additional methods for hypothesizing theoretical fits to the data base need to be developed. Existing digital elevation maps of the U.S. Geological Survey offer a particularly promising data base with sufficient accuracy for determining height, slope, and base distributions of terrain profiles. In the absence of surface truth or scattering data to the contrary and for situations in which surface shadowing and multiple scattering are negligible, an exponential distribution is recommended for the probability density function of the height random variable in statistical scattering models.

ACKNOWLEDGEMENT

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SECTION I

INTRODUCTION

Surface truth is defined as empirical data of a terrain or sea surface. The surface truth of particular interest in this paper is the surface profile distribution averaged over the same or similar ensemble of surface patches from which coherent and incoherent scatter measurements are averaged. This surface truth is useful in assessing statistical scattering models for multipath interference, terrain shadowing, and clutter.

One such statistical model, in which the surface height is assumed to be exponentially distributed, is of particular interest because this model was recently shown [1], to give a superior fit to three sets of experimental data for coherent forward scatter from smooth and moderately rough surfaces. The three sets of data consisted of microwave scatter from sea surfaces [2], microwave scatter from terrain and sea surfaces [3], and acoustical scatter from artificial surfaces submerged in a water tank [4].

In the model the surface profile is characterized with respect to the mean surface level (MSL) by an exponentially distributed height random variable which can be expressed as a function of two independent normally distributed base random variables (x and y components of the closest base distance to a zero height crossing on

the MSL) and an independent normally distributed angle tangent random variable. These surface profile random variables and distributions are defined in Appendix A.

The question arises as to how well this model agrees with the actual distributions of the surface profiles. It is found that empirical data of profile distributions is sparse and inconclusive.

The profile distributions associated with the scattering data of Beard (sea surfaces), Beckmann and Spizzichino (mostly terrain surfaces), and Boyd and Deavenport (model surfaces) are discussed in Sections 2, 3, and 4 respectively. Conclusions and recommendations are given in Section 5.

SECTION 2

SEA SURFACES AND THOSE ASSOCIATED WITH MICROWAVE SCATTERING DATA OF BEARD

Except for the standard deviation σ_H of the surface height, Beard does not give data for the sea surface profiles of his Gulf of Mexico and Golden Gate experiments. Beard does state that surface shadowing was not appreciable for most of his experimental data points even for rough sea states[2]. Sea surface statistics reported in the literature are reviewed in the remainder of this section.

Beard reports the cumulative distribution function $F_H(h)$ of water heights for artificially-generated waves in a water tank[5]. The original staircase data curve $F_N(h)$ is not shown, but processed data points for probabilities between 0.2% and 99.5% are plotted in Figure 1 on normal probability paper and are compared with normal and exponential cumulative distribution functions. Although a Kolmogorov-Smirnov test of fit is not given, the data points lie approximately on a straight line and are therefore better approximated by a normal distribution than an exponential distribution. However, similar measurements by Beard, for natural waves in the ocean, yield processed data points which have larger deviations from a straight line and therefore are not as well approximated by a normal distribution[6].

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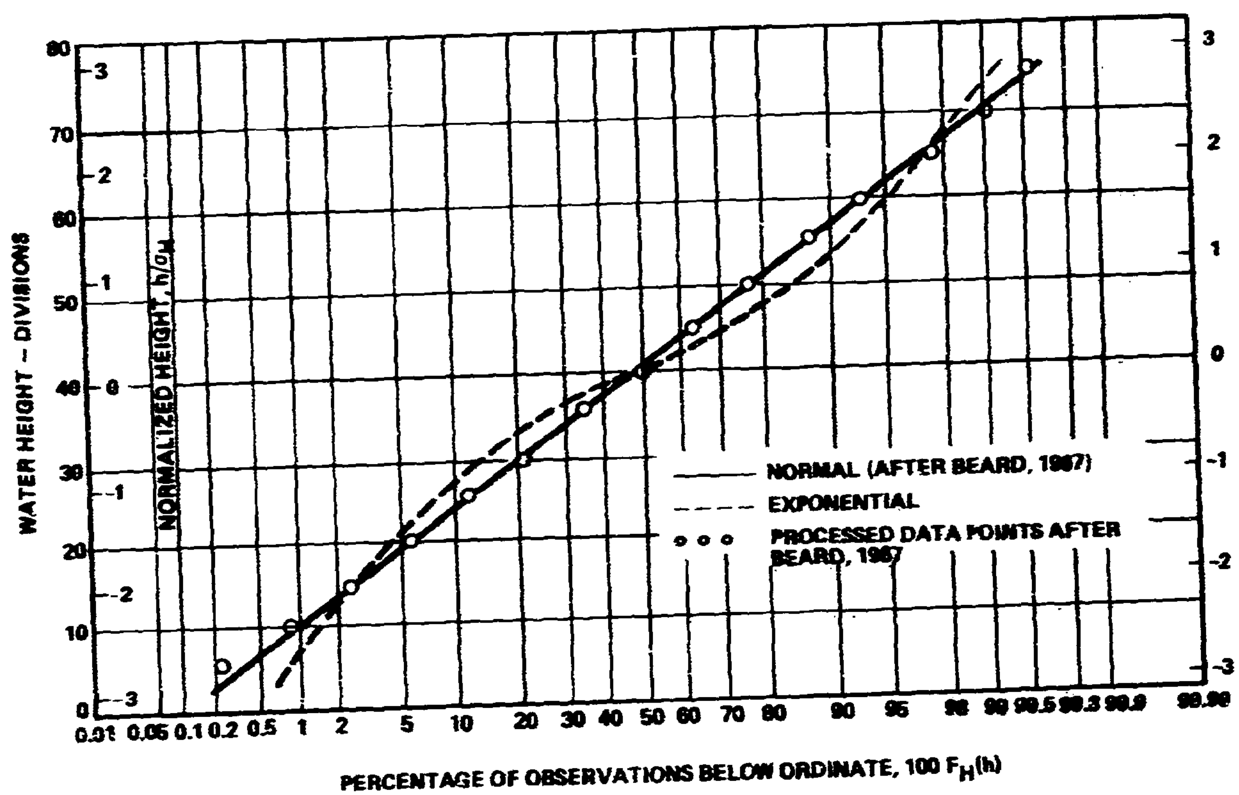

FIGURE 1. CUMULATIVE DISTRIBUTION OF WATER HEIGHTS FOR ARTIFICIALLY-GENERATED WAVES IN A WATER TANK. 450 READINGS AT 2-SECOND INTERVALS; σ_H = 0.106 CM; 12.92 DIVISIONS = $1\sigma_H$.

Histograms of the height, angle of pitch, and angle of roll of sea waves in a 23-knot wind are given by Longuet-Higgens et al[7] (see Figure 2). The histograms are compared with best fit zero-mean normal distributions whose standard deviation and bin size are different from the histograms but are in the same ratio as for the histograms. The height histogram is also compared with an exponential distribution whose standard deviation and bin size are the same as for the normal distribution. The histograms give the percentage $p_X(x)$ of data points with values between x and x + Δx of the random variable X for each interval Δx over the measured range of values. The cumulative distribution function $F_X(x)$ and probability density function $f_X(x)$ are related to $p_X(x)$ by

$$p_X(x) = F_X(x + \Delta x) - F_X(x) = \int_x^{x+\Delta x} f_X(t)\, dt$$

$$\approx f_X(x)\, \Delta x \ , \ f_X(x) \approx f_X(x + \Delta x) \qquad (1)$$

In Figure 2, the height random variable appears to be approximated visually as well by the exponential distribution as by the normal distribution. The slope random variables appear to be visually well-approximated by a normal distribution although other distributions might give as good a fit. However, visual fits are not necessarily a good indication of statistical fit. For example, on the basis of 2,000 observations, the probability that a normal distribution fits either the height or slope histograms of Figure 2 is appreciably less

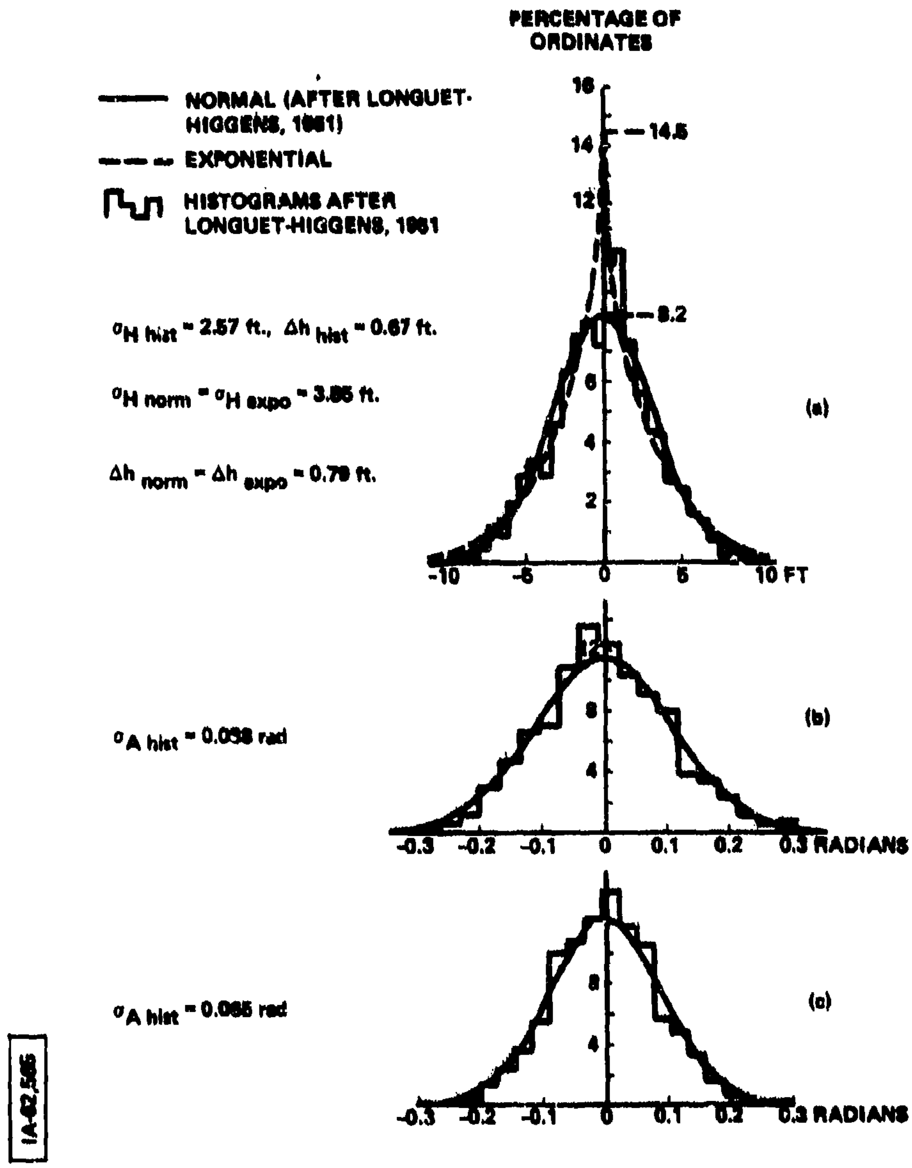

FIGURE 2. HISTOGRAMS OF THE SEA SURFACE IN A WIND OF 23 KNOTS. (a) HEIGHT, (b) ANGLE OF PITCH (CROSSWIND), (c) ANGLE OF ROLL (UPWIND)

than 1% utilizing a chi-squared test of fit.

Distributions of sea surface displacement have also been determined by Kinsman[8]. Kinsman concluded that the empirical frequency distribution of the height random variable is visually well-approximated by a non-zero mean normal distribution even though a Gram-Charlier distribution (an orthogonal polynomial series expansion in which the first term is the normal distribution) had a chi-squared test probability of fit of only 2.1% on the basis of approximately 12,000 data points. Kinsman's empirical frequency distribution[8] does not show the original histograms so that comparisons with theoretical probability density functions are inconclusive.

Prior to the findings of Longuet-Higgens et al, Cox and Munk[9] found that the slope of the sea surface is approximately normally distributed with zero mean in the cross-wind direction and with a positive mean in the downwind direction. The original histograms are not shown so that it is difficult to assess whether other distributions might give as good a fit to the data.

In summary of sea surface truth, Beard does not give profile distribution data for his Gulf of Mexico and Golden Gate experiments. The sea surface height histogram of Longuet-Higgens et al is well-approximated visually by either an exponential or a normal distribution. Sea slope data of several investigators are well-approximated visually by zero mean and non-zero mean normal distributions depending

upon wind direction although other distributions might give as good a fit. The chi-squared test probability that a normal distribution fits either the referenced sea height or sea slope data is, in most cases, appreciably less than 1%. No data was found for distributions of the base random variable.

SECTION 3

TERRAIN SURFACES AND THOSE ASSOCIATED WITH MICROWAVE SCATTERING DATA OF BECKMANN AND SPIZZICHINO

Beckmann and Spizzichino[3] summarize the data by several investigators of microwave coherent scatter from terrain and sea surfaces. The surface profile distributions associated with these experiments are not given although, in some cases involving a single propagation path over a fixed terrain surface, the deterministic surface profile is reported[10]. Since coherent and incoherent scatter are defined as statistical averages of signals received from an ensemble of patches (where a "patch" is the total surface area illuminated by the incident radiation at a given instant of time), a single patch profile is not of particular interest.

An extensive study of terrain profiles in the United States by Anderson[11], with corresponding transmission loss measurements by Miles and Barsis[12], are utilized by Longley and Rice[13] in a computer method for statistical prediction of tropospheric transmission loss. Cumulative distributions of the logarithm of interdecile height $\Delta h(d)$ as a function of path distance d are plotted on normal probability paper[13]. The interdecile height is the range in heights between 10% and 90% values of the cumulative distribution function $F_H(h)$ for the height deviations from the mean surface level of a given profile. The cumulative distribution function

$F_{\Delta H}[\Delta h(d)]$ of the interdecile height for different values of d is plotted in Figure 3 for 101 random paths within the United States. Longley and Rice found that the median values $\Delta h(d)_{med}$, corresponding to $F_{\Delta H}[\Delta h(d)_{med}] = 0.5$, increase monotonically with d and is given approximately by

$$\Delta h(d) = \Delta h(\infty)\ [1 - 0.8 \exp(-0.02d)]\ (m) \qquad (2)$$

where

$$\Delta h(\infty) \equiv \Delta h = \text{extrapolated value of } \Delta h \text{ for } d = \infty$$

$$d = \text{path distance in km}$$

Eq. (2) was found to be approximately valid also for 216 U.S. plains paths and 216 U.S. mountain paths. For the 101 U.S. random paths,

$$\Delta h(\infty) = \Delta h = 90 \text{ m}.$$

In Figure 3, the empirical values of interdecile height $\Delta h(d)$ appear to be approximately log normally distributed (i.e., $F_{\log\Delta H}[\log \Delta h(d)]$ is approximately a straight line) although other logarithmically distributed functions might give as good a fit. The cumulative distribution function $F_H[h(d)]$ of the height random variable H is not readily inferred from Figure 3 because $F_H[h(d)]$ is generally not equal to $F_{\Delta H}[\Delta h(d)]$. The interdecile height $\Delta h(d)$ is related to the standard deviation $\sigma_H(d)$ for normal and exponential distributions of the height random variable H, by

$$\Delta h(d) = \begin{cases} 2.6\ \sigma_H(d)\ , & H \text{ normally distributed} \\ 2.3\ \sigma_H(d)\ , & H \text{ exponentially distributed} \end{cases} \qquad (3)$$

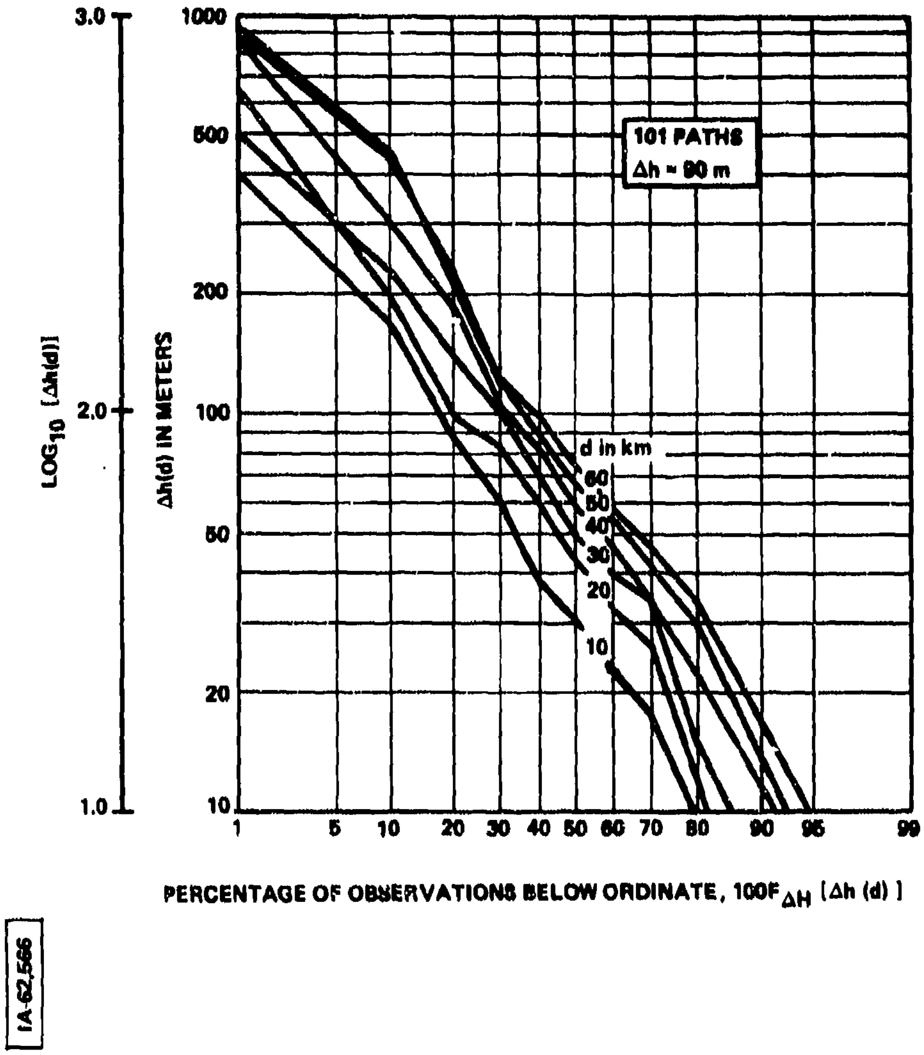

FIGURE 3: CUMULATIVE DISTRIBUTIONS OF INTERDECILE HEIGHT FOR 101 U.S. RANDOM PATHS (AFTER LONGLEY AND RICE, 1968).

In the Longley-Rice propagation program, the interdecile height is related to the standard deviation $\sigma_H(d)$, within the limits of the first Fresnel zone, by[13]

$$\Delta h(d) = \begin{cases} 2.56\ \sigma_H(d)\ , & \Delta h(d) \leq 4\ \mathrm{m} \\ 1.28\ \sigma_H(d)\ \exp\left\{0.5\ [\Delta h(d)]^{1/4}\right\}\ , & \Delta h(d) > 4\ \mathrm{m} \end{cases} \qquad (4)$$

Although histograms of terrain profile distributions are not readily found in the literature, the source data for determining such histograms do exist. For example, the U.S. Defense Mapping Agency has digital tapes of cartographic terrain maps, of $1^\circ \times 1^\circ$ map size on a scale of 1:250,000, with a bare ground vertical inaccuracy of ± 30 m maximum for an arbitrary point on the map[14]. The U.S. Geological Survey has two sets of digital elevation model (DEM) tapes of photogrammetric terrain maps on a scale of 1:24,000, digitized every 30 m horizontally, with a range of vertical inaccuracies of 0-7 m rms and 8-15 m rms[15]. Other potential data sources include the National Geographic Society and the Swiss Federal Mapping Agency. An accurate determination of profile distributions from many of the digital tapes of terrain maps may not be possible because the vertical accuracy of the maps is comparable to or larger than the standard deviation σ_H of the distributions. Assessments of terrain map accuracy and determination of terrain profile distributions have been performed by MIT Lincoln Laboratory as part of the cruise missile program but these results have not yet been released[16]. Digital

topographic maps of the Defense Mapping Agency have been utilized by the Rome Air Development Center (RADC) to derive height profile and surface shadowing distributions in modeling electromagnetic scattering from rough terrain(17). The RADC study found that the heights were exponentially distributed for the particular terrain which was investigated. A statistical method for hypothesis testing the fit of theoretical distributions to empirical data has also been reported by RADC(18),(19).

In summary of terrain surface truth, empirical distributions of the height, slope, or base random variables are neither given by Beckman and Spizzichino nor readily found in the literature.

SECTION 4

MODEL SURFACES ASSOCIATED WITH ACOUSTICAL SCATTERING DATA OF BOYD AND DEAVENPORT

The four sets of acoustical scattering data by Boyd and Deavenport are from four models whose surfaces are identical except for different scaling ratios (1:1, 2:1, and 4:1 vertical scaling with 1:1 horizontal scaling; 1:1 vertical scaling with 2:1 horizontal scaling). The models are scaled up from aeromagnetic intensity maps of a portion of the Canadian Shield[20]. Each set of acoustical scattering data was obtained by averaging the signals received on successive illuminations of different portions of a model surface. Histograms of the surfaces were determined by Mitchell[21]. Smooth curve probability density functions were fitted by Welton et al[22] to the histogram for the model with the roughest surface (4:1 vertical scaling). Welton concluded that all of the probability density functions (non-zero mean Edgeworth, zero-mean normal, and non-zero mean modified exponential) gave reasonable visual fits to the histrogram.

In Figure 4, this non-zero mean histogram is compared with zero-mean exponential and normal probability density functions $f_H(h)$ with the same standard deviation σ_H as the histogram. The overall visual fit of the distributions to the histogram are about equal--the exponential distribution giving a better fit to the peak

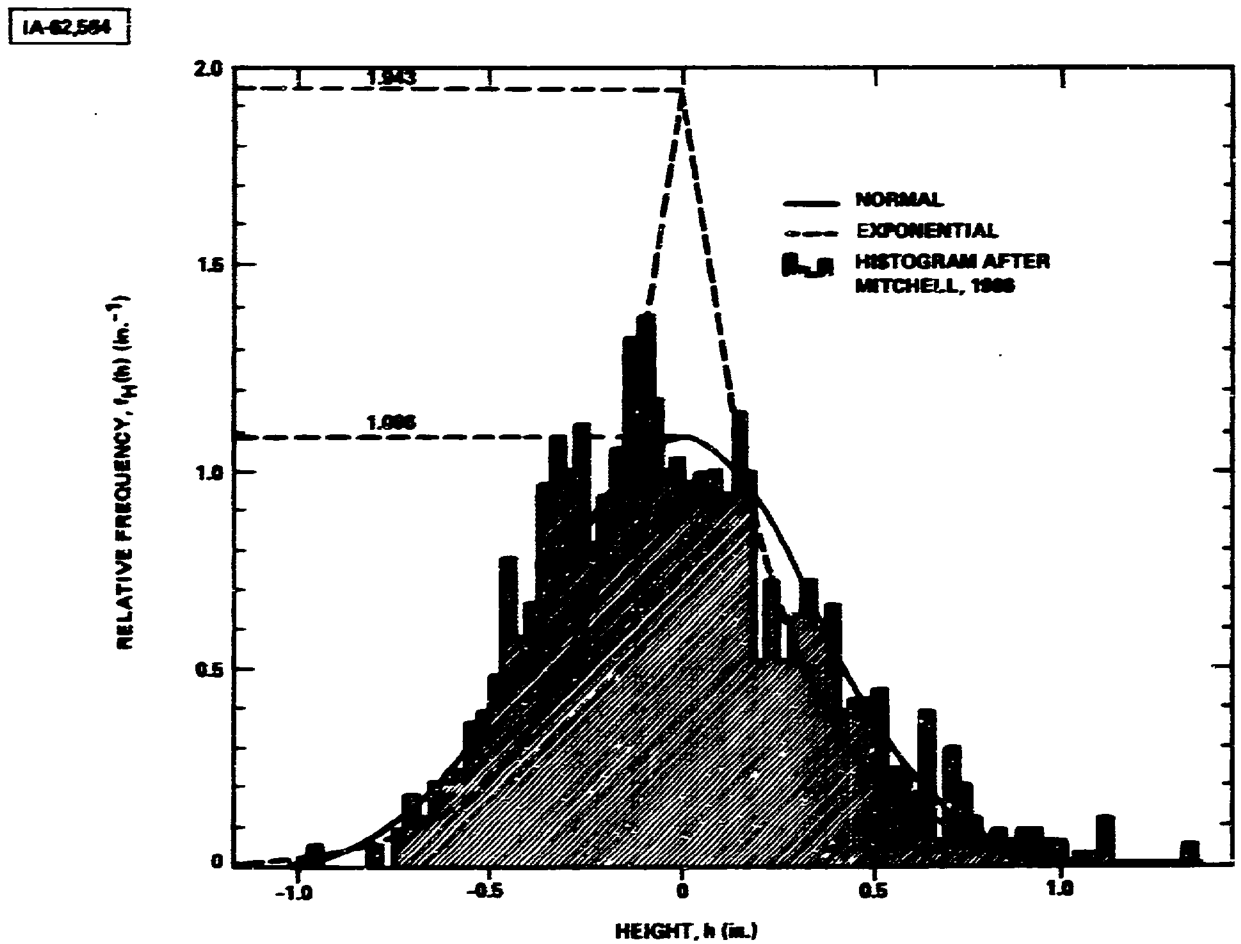

FIGURE 4. HISTOGRAM OF HEIGHTS MEASURED AT 1089 SAMPLE POINTS FROM A MODEL SURFACE. STANDARD DEVIATION σ_H = 0.364 INCHES.

and tails of the histogram and the normal distribution giving a better fit for intermediate values of the height random variable.

Data points for the height autocorrelation functions along the x and y axes of the roughest surface are also given by Mitchell[21]. A non-analytic (exponential) expression was found by Welton et al[22] to give only a slightly better fit to the data points than an analytic expression for the autocorrelation function.

In summary of artificial model surface truth, exponential and normal probability density functions give approximately equal overall visual fits to the height histogram for one of the model surfaces. Although empirical distributions of the slope and base random variables are not given, empirical height autocorrelation functions are given and are fit approximately equally well by an exponential expression and an expression whose derivative is well defined at all points.

SECTION 5

CONCLUSIONS AND RECOMMENDATIONS

The terrain and sea surface truth of profile distributions, averaged over the same or similar ensemble of surface patches for which coherent and incoherent scatter measurements are averaged, is of interest in assessing statistical scattering models for multipath interference, terrain shadowing, and clutter return. Distributions of surface height, slope, and base random variables are pertinent and interrelated.

The surface profile data, associated with three sets of experimental data for microwave and acoustical scatter, was found to be either nonexistent or inconclusive. For two sets of data for microwave scatter from terrain and sea surfaces, no empirical profile distributions are given. For the set of data for acoustical scatter from artificial model surfaces, a height histogram is given. However, both normal and exponential distributions give reasonable visual fits to the histogram.

The surface profile distributions which are reported in the literature are sparse and inconclusive. No terrain distributions were found but two references to recent determinations of terrain distributions from digital tapes of terrain maps were found. In one set of sea surface histograms, normal and exponential distributions were found to give equally good visual fits to the

height histogram whereas a normal distribution was found to give a good visual fit to the slope histogram. Other distributions might possibly give as good a fit. No histograms of the base random variables were found in the literature.

A surface profile data base and additional methods for hypothesizing fits to the data base need to be developed. Existing digital elevation maps of the U.S. Geological Survey offer a particularly promising data base with sufficient accuracy for determining height, angle tangent, and base distributions of terrain profiles. The interrelationship of these distributions offers an additional hypothesis for testing fits of proposed theoretical distributions to the data base. For example, if exponential, normal, and Rayleigh distributions give reasonable fits to the height, angle tangent, and base histograms respectively, then such a set of distributions are consistent provided that the angle tangent and base random variables are assumed to be independent random variables (see No. 6, Table A-1 of Appendix A).

Statistical scattering data from an ensemble of surfaces offers a potentially sensitive method of deducing surface profile truth provided that the statistical scattering model which is assumed is correct. Coherent scattering data from smooth and moderately rough surfaces suggests that terrain and sea surface heights are approximately exponentially distributed provided that one accepts a theoretical scattering model which assumes negligible surface

shadowing and multiple scattering[1]. In the absence of surface truth or scattering data to the contrary and for situations in which surface shadowing and multiple scattering are negligible, an exponential distribution is suggested for the probability density function of the height random variable in statistical scattering models.

LIST OF REFERENCES

1. M.W. Weiner and G.A. Robertshaw, "Coherent Scatter of Microwaves from MOderately Rough Surfaces", ESD-TR-81-147, The MITRE Corporation, Bedford, Mass., August 27,1981, AD A106133.

2. C.I. Beard, "Coherent and Incoherent Scattering of Microwaves from the Ocean", IRE Transactions on Antennas and Propagation, Vol. AP-9, pp. 470-483 (September 1961).

3. P. Beckmann and A. Spizzichino, "The Scattering of Electromagnetic Waves from Rough Surfaces", (Pergamon Press, Oxford, 1963) p. 318.

4. M.L. Boyd and R.L. Deavenport, "Forward and Specular Scattering from a Rough Surface: Theory and Experiment", J. Acoustical Society of America, Vol. 53, No. 3, pp. 791-801 (March 1971).

5. C.I. Beard, "Behavior of Non-Rayleigh Statistics of Microwave Forward Scatter from a Random Water Surface", IEEE Transactions on Antennas and Propagation, Vol. AP-15, pp. 649-657, Sept. 1967.

6. Private communication from C.I. Beard. Measurements of height distributions for natural waves in the ocean were made in the early 1950's at the Electrical Engineering Laboratory of the University of Texas but unfortunately the reports containing these results are not readily available.

7. M.S. Longuet-Higgens, D.E. Cartwight, and N.D. Smith, "Observations of the Directional Spectra of Sea Waves Using the Motions of a Floating Buoy", National Academy of Sciences, Ocean Wave Spectra. Proceedings of a conference, Easton, Maryland, May 104, 1961 (Prentice-Hall, Englewood Cliffs, N.J., 1963), pp. 111-136. In Table 2, the slope variances are 10^{-2} rad^2 (and not 10^{-2} deg^2) times the given numbers.

REFERENCES (Con't)

8. B. Kinsman, "Wind Waves: Their Generation and Propagation on the Ocean Surface" (Prentice-Hall, Englewood Cliffs, N.J., 1965), p. 344.

9. C. Cox and W. Munk, "Statistics of the Sea Surface Derived from Sun Glitter", J. Marine Research, Vol. 13, No. 2, pp. 198-227 (1954).

10. For example, see K. Bullington, "Reflection Coefficients of Irregular Terrain", Proc. IRE, Vol. 42, No. 8, pp. 1258-62 (August 1954).

11. The terrain profile study conducted by David Anderson for the Environmental Service Sciences Administration (ESSA), utilized existing terrain maps. (Telephone conversation with P. L. Rice on July 2, 1981).

12. M. J. Miles and A. P. Barsis, "Summary of 20-100 MHZ propagation measurement results over irregular terrain using low antenna heights", U.S. Department of Commerce, ESSA Technical Report IER 10-ITSA, 1966.

13. A. G. Longley and P. L. Rice, "Prediction of Tropospheric Radio Transmission Loss Over Irregular Terrain, A Computer Method - 1968", U.S. National Bureau of Standards, ESSA Technical Report ERL 79-ITS, 1968, Annex 2, pp. 2-1 - 2-42. See also Annex 3, p. 3-4.

14. Telephone conversation on June 25, 1981 with Mr. Dennis Granato, Digital Requirements to Support Weapons Systems and Simulators, Plans and Requirements Division, U.S. Defense Mapping Agency.

15. Telephone conversation on July 6, 1981 with Mr. Arthur Woll, Digital Products, National Cartographic Information Center, U.S. Geological Survey, Reston, Virginia.

16. Telephone conversation on June 25, 1981 with R. Francois of MIT Lincoln Laboratory.

17. R. J. Papa, J. F. Lennon, and R. L. Taylor, "Prediction of Electromagnetic Scattering for Rough Terrain Using Statistical Parameters Derived from Digitized Topographic Maps", Rome Air Development Center, RADC-TR-80-289, September 1980, AD A094104.

18. J. F. Lennon and R. J. Papa, "Statistical Characterization of Rough Terrain", Rome Air Development Center, RADC-TR-80-9, February 1980, AD A087746.

19. J. F. Lennon and R. J. Papa, "An Error Analysis Technique for Statistical Hypothesis Testing", Rome Air Development Center, RADC-TR-81-8, 1981, AD A098104.

20. C. W. Horton, W. B. Hampkins, and A. A. J. Hoffman, "A Statistical Analysis of Some Aerodynamic Maps from the Northwestern Canadian Shield", Geophysics, Vol. 29, pp. 582-601 (1964).

21. S. K. Mitchell, "Underwater Sound Scattering from an Irregular Surface", Master of Arts Thesis, Physics Department, University of Texas, Austin, Texas, 1966.

22. P. J. Welton, H. G. Frey, and P. Moore, "Experimental Measurements of the Scattering of Acoustic Waves by Rough Surfaces", J. Acoustical Society of America, Vol. 52, No. 5 (Part 2), pp. 1553-1563 (1972).

23. H. Van Trees, Probability Study Guide, Block II "Random Variables" (MIT Center for Advanced Engineering Study, Catalog No. 12-2200, Cambridge, Mass., 1970).

24. W. B. Davenport, Jr., "Probability and Random Processes" (McGraw-Hill, N.Y., 1970).

25. I. S. Gradshteyn and I. M. Ryzhik, "Tables of Integrals, Series, and Products" (Academic Press, N.Y., 1965).

APPENDIX A

SURFACE PROFILE RANDOM VARIABLES AND DISTRIBUTIONS

A three-dimensional rough surface is characterized by a mean surface level (MSL) and a surface profile with regions which extend above the MSL (labeled +), below the MSL (labeled -), and on the MSL (labeled 0), as shown in Figure A-1. The point $Q(x_1, y_1, 0)$ is the zero height crossing closest to the projection of the point $P(x, y, z)$ onto the MSL. The height H, angle tangent $A = \mathrm{TAN}\ \alpha$, and base B_x, B_y random variables with values h, $\tan \alpha$, b_x, and b_y respectively are defined in Figure A-1. The point $P(x,y,z)$ is at a height $h(x,y)$, with respect to the MSL, given by

$$h(x,y) = z = b \tan \alpha \qquad \text{(A-1)}$$

where $b = (b_x^2 + b_y^2)^{1/2}$.

The height random variable H is therefore related to the base and angle tangent random variables by

$$H = BA \qquad \text{(A-2)}$$

where $B = (B_x^2 + B_y^2)^{1/2}$.

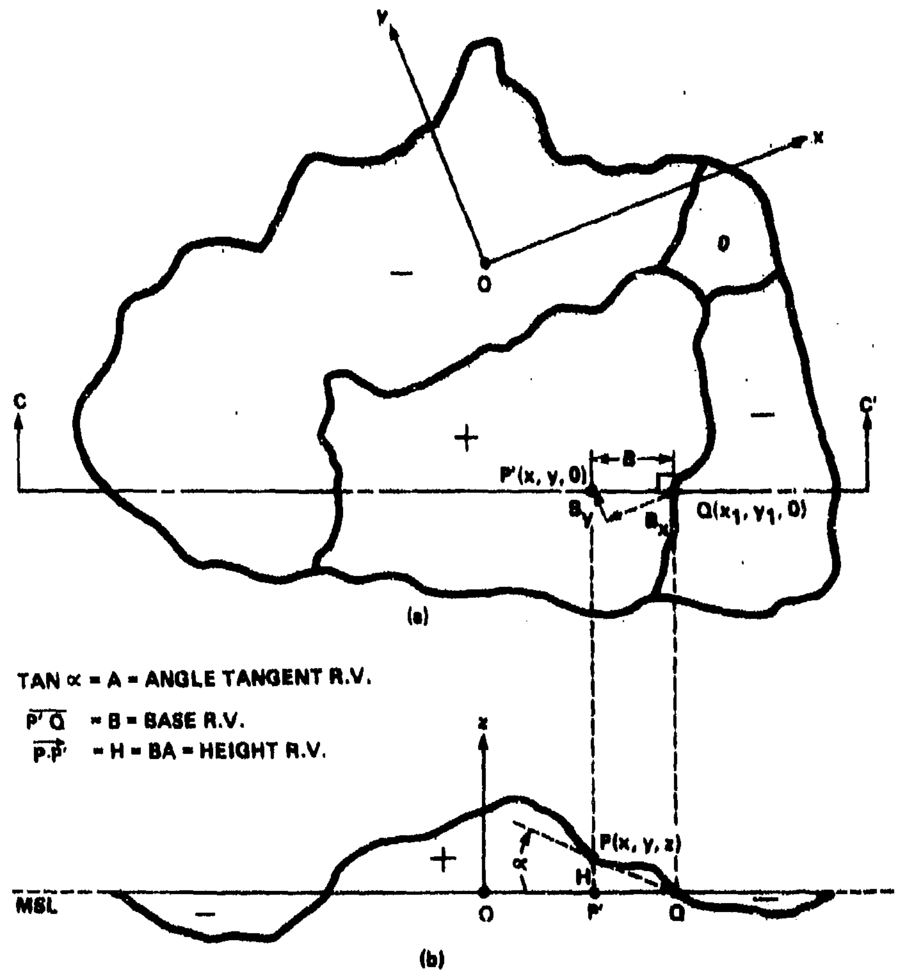

FIG. A-1. SURFACE PROFILE RANDOM VARIABLES
(a) Plan View
(b) Elevation View (Section C-C')

This set of random variables is one of several possible sets of random variables that can characterize a surface profile. For example, a set consisting of height, slope, and base intercept random variables could also be defined. The former set is attractive because the mean and variance of each random variable is most likely finite whereas in the latter set the variance of the base intercept random variable is most likely infinite. On the other hand, if empirical data of the slope random variable is available, then the latter set may be preferable.

Since the random variables comprising a set are geometrically related by expressions such as Eq. (A-2), the probability density functions of these variables are also related to each other. These relationships are a function of the statistical interdependence of the random variables. The statistical interdependence, in turn, is a function of the random process associated with the surface profile. Although the random process is generally not known, it is possible to tabulate the relationship between probability density functions for different assumptions concerning the statistical independence of the random variables. Some of these relationships are given in Table A-1 for a single output random variable as a function of two input random variables. It should be noted in Table A-1 (compare Nos. 6, 9-11) that if $f_Z(z)$ is the derived probability density function

TABLE A-1.

PROBABILITY DENSITY FUNCTIONS FOR A SINGLE OUTPUT RANDOM VARIABLE Z A

NO.	TRANSFORMATION, Z	INPUT JOINT DENSITY, $f_{XY}(x,y)$	
1	Z = X + Y	ARBITRARY	$\int_{-\infty}^{\infty}$
2	Z = Y − Y	ARBITRARY	$\int_{-\infty}^{\infty}$
3	Z = X Y	ARBITRARY	$\int_{-\infty}^{\infty}$
4	Z = Y/X	ARBITRARY	$\int_{-\infty}^{\infty}$

TABLE A-1.

PUT RANDOM VARIABLE Z AS A FUNCTION OF TWO INPUT RANDOM VARIABLES X AND Y

	OUTPUT DENSITY, $f_Z(z)$	REFERENCE
	$\int_{-\infty}^{\infty} f_{XY}(x, z-x)\, dx$	24, PROBLEM 2.14.3
	$\int_{-\infty}^{\infty} f_{XY}(x, x-z)\, dx$	24, PROBLEM 2.14.3
	$\int_{-\infty}^{\infty} \frac{f_{XY}(x, z/x)}{\lvert x\rvert}\, dx$	24, PROBLEM 2.14.3
	$\int_{-\infty}^{\infty} \lvert x\rvert\, f_{XY}(x, xz)\, dx$	24, PROBLEM 2.14.3

TABLE A-1 (Concluded)

IN NOS. 5-11, X and Y ARE ASSUMED TO BE STATISTICALLY INDEPENDENT RANDOM VARIABL

NO.	TRANSFORMATION, Z	INPUT JOINT DENSITY, $f_{XY}(x,y)$	
5	$Z = (X^2 + Y^2)$ X, NORMAL, $m_X = 0$ Y, NORMAL, $m_Y = 0$ $\sigma_X = \sigma_Y = \sigma$	$f_X = (2\pi\sigma^2)^{-1/2} \exp(-x^2/2\sigma^2)$, $f_Y = (2\pi\sigma^2)^{-1/2} \exp(-y^2/2\sigma^2)$,	$(z/\sigma^2) \exp(-z^2/$ RAYLEIGH
6	$Z = X\,Y$ X, NORMAL, $m_X = 0$ Y, RAYLEIGH	$f_X = (2\pi\sigma^2)^{-1/2} \exp(-x^2/2\sigma_X^2)$ $f_Y = \begin{cases} (\sigma_Y)^{-2}\, y \exp(-y^2/2\sigma_Y^2), & y \geq 0 \\ 0, & y < 0 \end{cases}$	$(2\sigma_Z^2)^{-1/2}$ exp (EXPONENTIAL
7	$Z = X\,Y$ X, RAYLEIGH Y, ARC SIN	$f_X = \begin{cases} (x/\sigma^2) \exp(-x^2/2\sigma^2), & x \geq 0 \\ 0, & x < 0 \end{cases}$ $f_Y = \begin{cases} 1/(\pi\sqrt{1-y^2}), & -1 < y \leq 1 \\ 0, & \text{elsewhere} \end{cases}$	$(2\pi\,\sigma^2)^{-1/2}$ exp NORMAL
8	$Z = Y/X$ X, NORMAL, $m_X = 0$ Y, NORMAL, $m_Y = 0$	$f_X = (2\pi\,\sigma_X^2)^{-1/2} \exp(-x^2/2\sigma_X^2)$ $f_Y = (2\pi\sigma_Y^2)^{-1/2} \exp(-y^2/2\sigma_Y^2)$	$\frac{1}{\pi} \cdot \frac{\sigma_X/}{1 + [(}$ CAUCHY when
9	$Z = Y/X$ X, RAYLEIGH Y, NORMAL, $m_Y = 0$	$f_X = \begin{cases} (x/\sigma^2) \exp(-x^2/2\sigma^2), & x \geq 0 \\ 0 & x < 0 \end{cases}$ $f_Y = (2\pi\sigma_Y^2)^{-1/2} \exp(-y^2/2\sigma_Y^2)$	$\frac{1}{2} \cdot \frac{\sigma_X}{[1 + (}$
10	$Z = Y/X$ X, RAYLEIGH Y, EXPONENTIAL	$f_X = \begin{cases} (x/\sigma^2) \exp(-x^2/2\sigma^2), & x \geq 0 \\ 0, & x < 0 \end{cases}$ $f_Y = (2\sigma_Y^2)^{-1/2} \exp(-\sqrt{2}\,\lvert y \rvert/\sigma_Y)$	$-\lvert z \rvert\ (\sigma_X^2/\sigma_Y^2)$ SEE NOT
11	$Z = Y/X$ X, NORMAL, $m_X = 0$ Y, EXPONENTIAL	$f_X = (2\pi\sigma_X^2)^{-1/2} \exp(-x^2/2\sigma_X^2)$ $f_Y = (2\sigma_Y^2)^{-1/2} \exp(-\sqrt{2}\,\lvert y \rvert/\sigma_Y)$	$\frac{1}{\sqrt{\pi}}\ \frac{\sigma_X}{\sigma_Y} -$ | SEE NOT

NOTE 1. erfc (x) = 1

TABLE A-1 (Concluded)

DEPENDENT RANDOM VARIABLES SO THAT $f_{XY}(x, y) = f_X(x)\, f_Y(y)$

OUTPUT DENSITY, $f_Z(z)$	REFERENCE
$(z/\sigma^2)\exp(-z^2/2\sigma^2)$, $z \geq 0$; 0, $z < 0$, $\sigma_Z = \sqrt{2}\,\sigma$ RAYLEIGH	24, APPENDIX, TABLE C.3
$(2\sigma_Z^2)^{-1/2}\exp(-\sqrt{2}\,\lvert z\rvert/\sigma_Z)$, $\sigma_Z = \sqrt{2}\,\sigma_X\ \sigma_Y$ EXPONENTIAL	1, APPENDIX 25, p. 195-196.
$(2\pi\sigma^2)^{-1/2}\exp(-z^2/2\sigma^2)$ NORMAL	25, p. 195
$\frac{1}{\pi} \cdot \frac{\sigma_X/\sigma_Y}{1 + [(\sigma_X/\sigma_Y)z]^2}$	24, APPENDIX, TABLE C.3
CAUCHY when $\sigma_X = \sigma_Y$ $\frac{1}{2} \cdot \frac{\sigma_X/\sigma_Y}{[1 + (\sigma_X/\sigma_Y)^2 z^2]^{3/2}}$	4 AND 26, 3.461.2
$-\lvert z\rvert\,(\sigma_X^2/\sigma_Y^2) + 4\sqrt{\pi}\left(\frac{\sigma_X}{\sigma_Y}\right)^3 \cdot \left(\frac{\lvert z\rvert^2}{4} + \frac{\sigma_Y^2}{8\sigma_X^2}\right)\exp\left(\frac{\lvert z\rvert^2\,\sigma_X^2}{\sigma_Y^2}\right)\mathrm{erfc}\left(\frac{\lvert z\rvert\sigma_X}{\sigma_Y}\right)$ SEE NOTE 1	4 AND 26, 3.462.7
$\frac{1}{\sqrt{\pi}}\ \frac{\sigma_X}{\sigma_Y} - \frac{\lvert z\rvert\sigma_X^2}{\sigma_Y}\exp(\lvert z\rvert^2\,\sigma_X^2/\sigma_Y^2)\,\mathrm{erfc}(\lvert z\rvert\,\sigma_X/\sigma_Y)$ SEE NOTE 1	4 AND 26, 3.462.5

NOTE 1. $\mathrm{erfc}(x) = 1 - \phi(x)$ where $\phi(x) = (2/\sqrt{\pi})\int_0^x \exp(-t^2)\,dt$

2

when X and Y are statistically independent and $[f_X(x), f_Y(y)]$ are specified, then it generally does <u>not</u> follow that $f_X(x)$ is the derived probability density function if Y and Z are statistically independent and $[f_Y(y), f_Z(z)]$ are specified even though the same geometrical relationship between X, Y, and Z is maintained.

Several relationships between products and quotients of two statistically independent random variables are given in Table A-1 for at least one of the random variables either normal or exponentially distributed.

The normalized probability density function $\sigma_X f_X(x)$ and cumulative distribution function $F_X(x) = \int_{-\infty}^{x} f_X(t)dt$ are plotted in Figures A-2 and A-3 respectively for normal and exponential distributions of the random variable X normalized to the standard deviation σ_X.

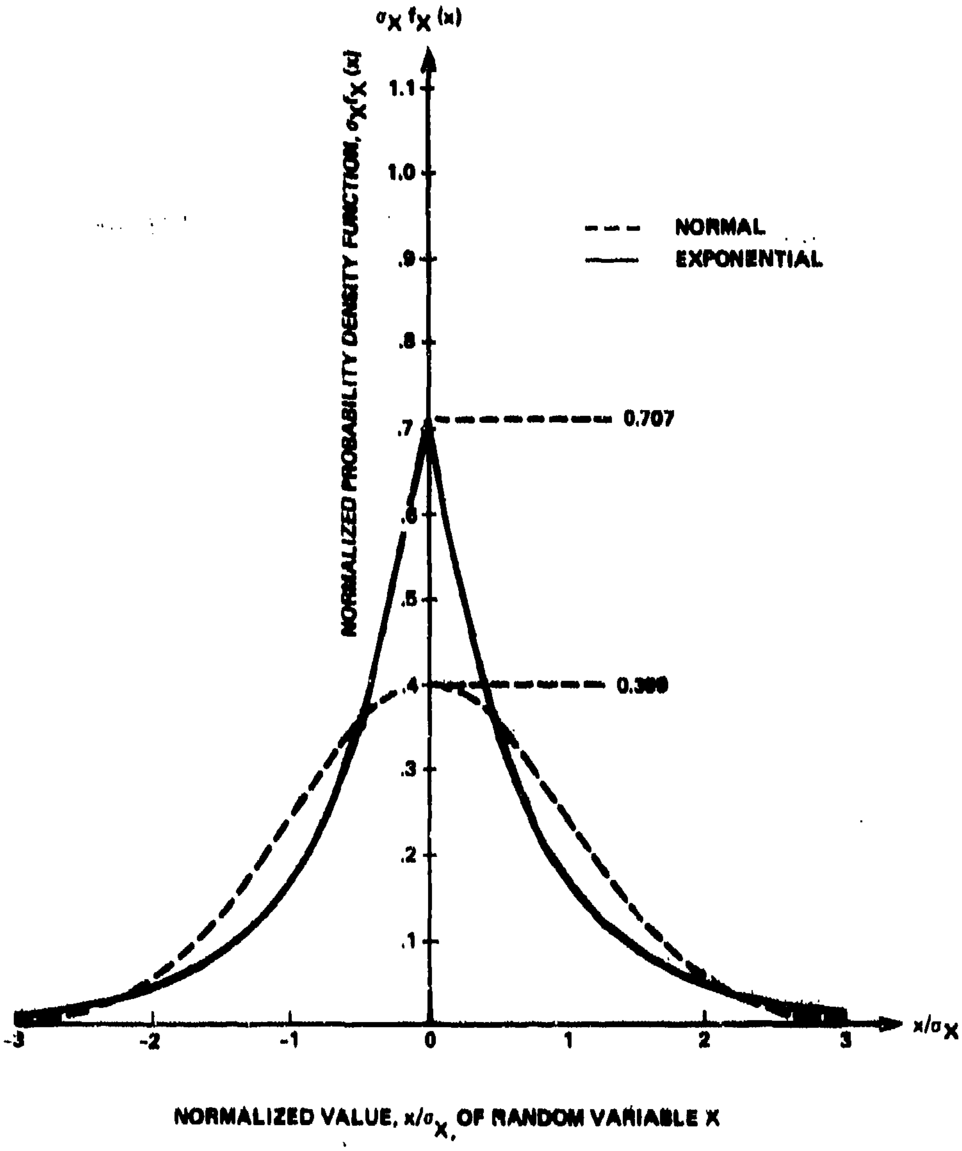

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FIG. A-2. PROBABILITY DENSITY FUNCTIONS FOR NORMALLY AND EXPONENTIALLY DISTRIBUTED RANDOM VARIABLES

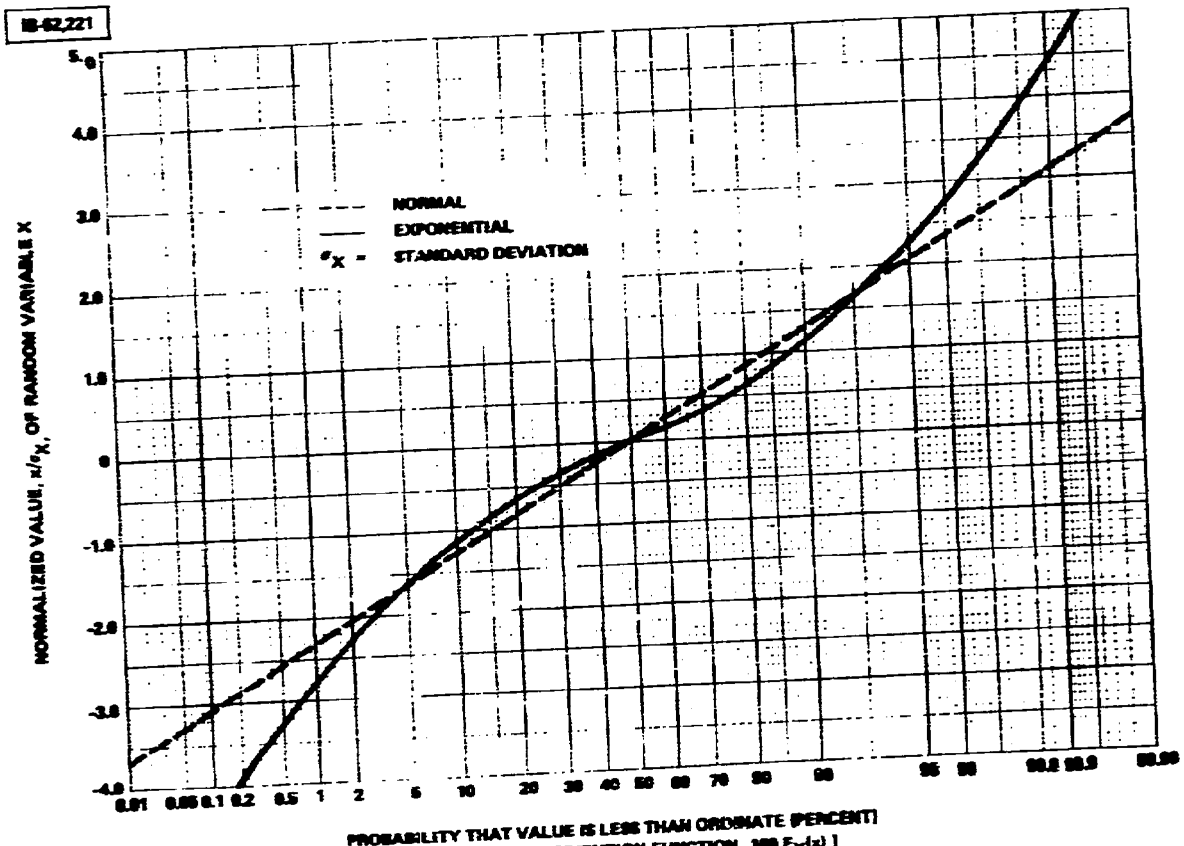

FIG. A-3. CUMULATIVE DISTRIBUTION FUNCTIONS, PLOTTED ON NORMAL PROBABILITY PAPER, FOR NORMALLY AND EXPONENTIALLY DISTRIBUTED RANDOM VARIABLES